OPTICAL PROPERTIES OF SECOND-ORDER CURVES IN LIGHTING ENGINEERING AND PARABOLIC ANTENNA SYSTEMS

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Abstract: *In this article, the optical properties of second-order curves, particularly the parabola, are analyzed. The ability of parabolic surfaces to convert spherical waves into plane waves is mathematically modeled using examples of car headlights, lighting technology, and satellite communication antennas. Additionally, practical metric problems related to the design of parabolic antennas are presented.*

Keywords: *parabola, paraboloid, optical property, focal parameter, spherical wave, offset antenna, lighting engineering, reflector.*

Second-order curves, particularly the rotational paraboloid, which is the spatial analogue of the parabola, play a central role in many technological achievements of humankind due to their unique geometric and optical properties. If in astronomical telescopes we utilize the property of gathering parallel light rays coming from infinity into a single point (the focus), the inverse of this process is also widely used in lighting engineering and radio technology: converting spherical waves emanating from the focal point into a parallel wave front. In this article, we analyze how this property of the parabola

is mathematically modeled and implemented in practice in lighting engineering and communication devices.

From a mathematical point of view, if a light or radio wave source is considered point-like, it emits spherical waves in all directions. To transmit waves over long distances without energy loss, it is required to convert the waves into a parallel beam in a specific direction. The most perfect geometric solution to achieve this is the rotational paraboloid.

Let the canonical equation of a parabola be given in the form $y^2 = 2px$. Its focal point is located at $F\left(\frac{p}{2}, 0\right)$. According to the definition of a parabola, the distances from each of its points to a given point (focus) and to a given line (directrix) are equal to each other. Rotating this curve in space generates a paraboloid of revolution. According to its optical property, if a light source is placed at the focal point F, any ray emanating from it will propagate parallel to the parabola's axis of symmetry after reflecting from the parabola's surface. To prove this, building upon previous proofs, we consider the relationship of tangents drawn from arbitrary points X and Y of the parabola with the focal point. Tangents drawn from any two points X and Y of the parabola intersect at a point P, and this point P is the center of the circumscribed circle of the triangle $VFX'Y'$. Here, X' and Y' are the orthogonal projections of points X and Y, respectively, onto the directrix. Furthermore, the projection P' of point P onto the directrix bisects the segment $X'Y'$. These geometric properties confirm that a wavefront emanating from the focus F, after reflecting from the paraboloid surface, forms a synchronous phase with the directrix (or planes parallel to it), thereby ensuring the strict parallelism of the light beam.

The operating principle of modern luminaires and car headlights is based on the paraboloid geometry mentioned above. The reflective surface (reflector) of a car headlight is mainly constructed in the shape of a paraboloid.

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Long-distance illumination (High beam): An incandescent filament (or LED source) is installed precisely at the focal point of the paraboloid. As a result, light rays reflect from the reflector, forming a strictly parallel and powerful beam along the road.

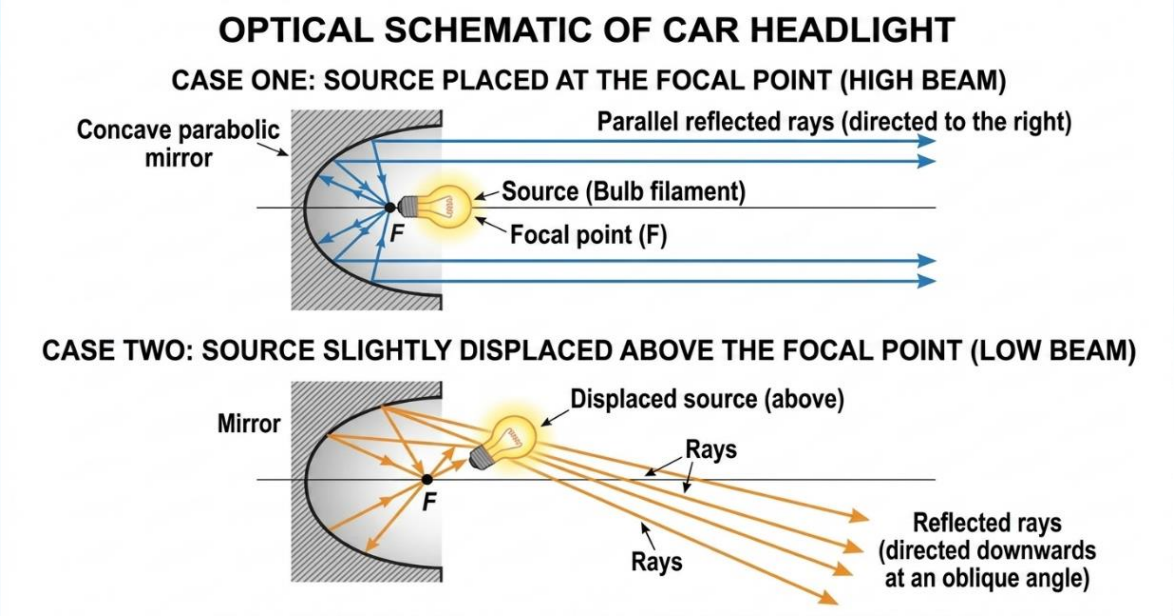


Figure 1

Short-distance illumination (Low beam): To avoid dazzling the eyes of an oncoming driver, the light beam should not be parallel but directed downwards. To achieve this, a second filament is installed in the bulb slightly in front of and above the focus. According to the laws of geometric optics, when the source is displaced, the reflected rays are no longer parallel but deviate at an angle to the axis (Figure 1).

Parabolic antennas: Communication and Radiotelescopes. Paraboloid surfaces are also ideal mirrors for radio waves. Radio waves arriving from very distant satellites reach Earth as a wave front parallel to the surface. A parabolic antenna collects these waves and concentrates all the energy at a single point — the focal point where the receiver is installed.

Offset Antennas: In classic symmetric antennas, the receiver lies in the center, which blocks a portion of the incoming waves. To overcome this, offset antennas were invented. An offset antenna is an asymmetric part cut from the

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side, not the central part, of a paraboloid of revolution. Optical and geometric laws are preserved, but the receiver (focus) now shifts outside the center of the "dish" and does not cast a shadow on the rays (Figure 2).

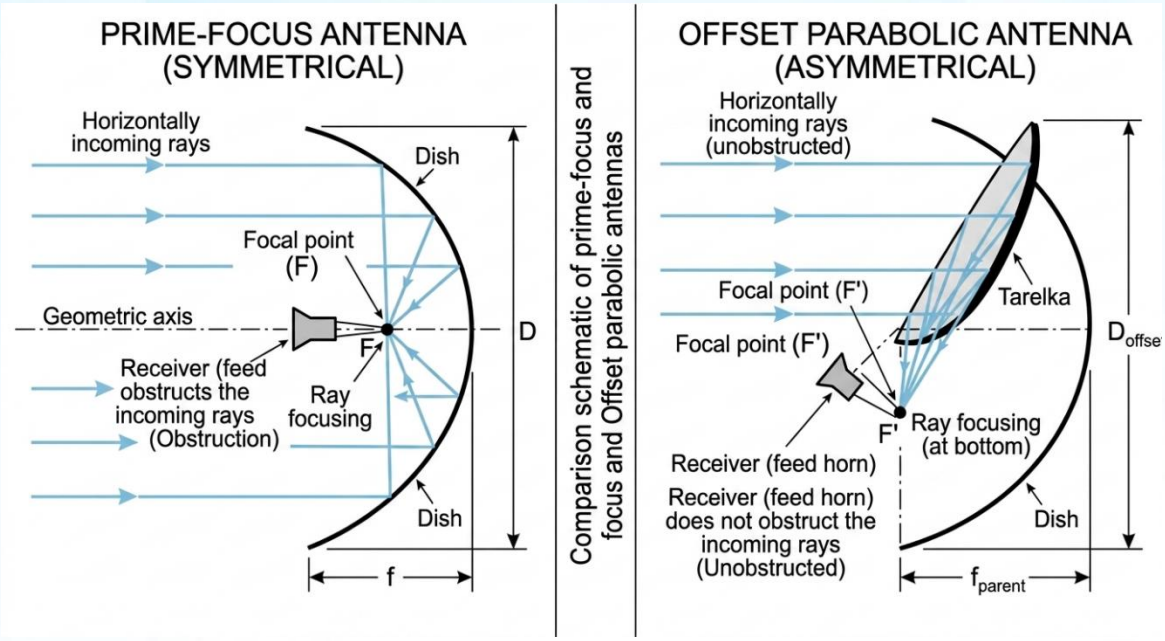


Figure 2

Mathematical modeling plays a key role in calculating the dimensions of parabolic surfaces. Below, we will analyze practical geometric problems.

Problem 1. A parabolic antenna is being manufactured. The antenna's diameter is $D=120$ cm, and its depth at the center is $h=15$ cm. At what distance f from the antenna's center (at the focus) should the signal-receiving converter be installed?

Solution: The vertex of the parabola is at the origin $(0,0)$, and its equation is taken in the form $x^2=2py$ or $x^2=4fy$. The coordinates of the antenna's edges will be $x=\frac{D}{2}=60$ cm and $y=h=15$ cm.

$$(60)^2=4\cdot f\cdot 15$$

$$3600=60f \Rightarrow f=\frac{3600}{60}=60 \text{ sm.}$$

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Answer: The converter must be installed exactly 60 cm away from the center.

Problem 2. The long-range reflector of an automobile headlight is a rotational paraboloid with a designated focal length of $f = 40$ mm. If engineers have confirmed that the full diameter of the headlight should be $D = 200$ mm, calculate the maximum depth h of this reflector.

Solution: For this problem, we solve the universal formula $f = \frac{D^2}{16h}$ for depth h:

$$h = \frac{D^2}{16f}$$

Substituting the values, we calculate:

$$h = \frac{200^2}{16 \cdot 40} = \frac{40000}{640} = 62.5 \text{ mm.}$$

Answer: The depth of the projector mirror will be 62.5 mm. This dimension ensures it fits ideally into the headlight housing.

Problem 3. A problem involving the mathematical analysis of the geometric properties of a parabola. The locus of all points P from which the parabola appears at a right angle is the directrix of that parabola. Furthermore, if lines PX and PY are tangents to the parabola at points X and Y respectively, then the line XY passes through the focus F, and the segment PF is the altitude of triangle ΔPXY. To prove this property, it is used that point P lies on the directrix of the parabola and that points X' and Y' are the orthogonal projections of points X and Y onto the directrix, respectively. According to the definition of a parabola and the SAS (Side-Angle-Side) congruence criterion for triangles, triangles ΔPXF and $\Delta PXX'$ are congruent. Therefore, $\angle PFX = \angle PX'X = 90^\circ$, from which the segment PF is the altitude of triangle ΔPXY. On the other hand, $\angle PFY = \angle PY'Y = 90^\circ$, and also the

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equality $\angle XPY = \frac{1}{2}(\angle FPX' + \angle FPY') = 90^{\circ}$ holds. These isogonal regularities are important in analyzing the dispersion of waves impinging from the edge angles of antennas.

The applications of parabolic properties, including those related to its directrix and tangents, are found in numerous fields. All parabolic surfaces in lighting technology and telecommunications are designed directly based on the laws of mathematics and analytical geometry. Forming parallel beams of light from a point source, and perfect reception of radio waves via offset antennas—all these are practical triumphs of ancient mathematical concepts, known as conic sections, in human technology.

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