MAXSUS YO'L BILAN YECHILADIGAN MASALALAR

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Bu mavzuga tegishli masalalarni yechish uchun kerak bo'ladigan nazariy ma'lumotlar matnli masalalarni yechishda deyarli tushuntirib o'tiladi. Faqat ayrim masalalarni yechish uchun juda foydali qo'shimcha ma'lumotlar bo'lishi mumkin. Ko'pgina tenglamalarda tenglikni ikki qismida 2 xil nomdagi funksiya qatnashadi. Masalan, trigonometrik funksiya va kvadrat funksiya yoki shunga o'xshash holatlar bo'ladi. Bunday tenglamalar ***transendent tenglamalar*** deyiladi va ko'pincha bunday tenglamalar berilganda tenglama nechta ildizga ega ekanligi so'raladi va bu savolga javob berish uchun ko'pincha grafik usuldan foydalaniladi. Ya'ni berilgan funksiyalarning grafiklarini bitta chizmada tasvirlab grafiklar necha marta kesishsa ildiz o'shancha bo'lishi aniqlanadi.

Grafik usuldan ham osonroq usul bo'lishi mumkin. Ba'zi tenglikning chap qismida turgan funksiyaning eng katta qiymati o'ng qismida turgan funksiyaning eng kichik qiymati bo'lishi mumkin. Agar shunday bo'lsa faqat argumentning o'sha shartni qanoatlantiruvchi qiymatini topish qoladi.

Ushbu mavzudagi va boshqa masalalarni yechishda quyidagi eslatmalar juda foydali bo'lishi mumkin.

Eslatma: Agar $A \ge 0,\ B \ge 0,\ C \ge 0$ bo'lsa, u holda $A+B+C=0$ tenglik faqat va faqat $A=0,\ B=0,\ C=0$ bo'lgandagina to'g'ri bo'ladi.

Misol: Agar $(a-|b|)^2+(a-2)^2=0$ bo'lsa, $2a-3b$ ning qiymatini toping.

$a-|b|=0,\ va\ \ a-2=0 \Rightarrow\ a=2$

$2-|b|=0,\ \ |b|=2,\ \ b=2,\ \ b=-2$

1) $2\cdot 2-3\cdot 2=-2$ 2) $2\cdot 2-3\cdot(-2)=10$ J: -2 va 10

Misol: $x^2+y^2+2(2x-3y)+|z-xy|+13=0$ bo'lsa, $x+y+z=?$

$x^2+y^2+2(2x-3y)+|z-xy|+13=0,$ $x^2+4x+4+y^2-6y+9+|z-xy|=0$

$(x+2)^2+(y-3)^2+|z-xy|=0,$ $x+2=0,\quad y-3=0,\quad z-xy=0$

$x=-2,\quad y=3,\quad z=3\cdot(-2)=-6 \Rightarrow z+x=y=-2+3-6=-5$ J:-5

Eslatma: Bizga $f(x)=g(x)$ tenglama berilganda shunday A sonni topish mumkin bo'lsaki, $f(x)\le A$ va $g(x)\ge A$ tengsizliklar o'rinli bo'lsa, u holda $f(x)=A$ va $g(x)=A$ tenglamalarning umumiy ildizini topish yetarli.

Misol: $2\cos x=2^x+2^{-x},\quad 2\cos x\le 2,\quad 2\cos x=2,\quad x=0$ da $2\cdot\cos 0=2$

$2^x+2^{-x}\ge 2,\quad 2^x+\frac{1}{2^x}=2,\quad 2^x=1,\quad 2^x=2^0,\quad x=0,\quad J: x=0$

Misol: $\cos(\lg(2-3^{x^2}))=3^{x^2}$ tenglamaning ildizi nechta?

$\cos(\lg(2-3^{x^2}))\le 1$ va $3^{x^2}\ge 1$

$\cos(\lg(2-3^{x^2}))=0,\quad x=0$ da $\cos(\lg(2-3^0))=1,\quad 3^{x^2}=1,\quad 3^{x^2}=3^0,\quad x=0$

Javob: ildizi bitta x=0.

Misol: $x\in[-3\pi;3\pi]$ oraliqda $\sin(\frac{\pi\sqrt{5}}{20}x)=21-4\sqrt{5}x+x^2$ tenglamaning nechta ildizi bor?

$\sin(\frac{\pi\sqrt{5}}{20}x)=21-4\sqrt{5}x+x^2,$ $21-4\sqrt{5}x+x^2=x^2-2\cdot 2\sqrt{5}+20+1=(x-2\sqrt{5})^2+1$

$\sin(\frac{\pi\sqrt{5}}{20}x)\le 1,$ $(x-2\sqrt{5})^2+1\ge 1 \Rightarrow x=2\sqrt{5}$ bo'lganda $\sin\frac{\pi}{2}=1$

Javob: bitta ildizi bor.

Eslatma: Bir o'zgaruvchili n-darajali ko'phad

$P(x)=ax^n+bx^{n-1}+cx^{n-2}+\ldots+t(x)+r$

ko'rinishida yoziladi. Bunda $a,b,c,\ldots,t,r$ berilgan sonlar bo'lib, ko'phadning koeffisentlari deyiladi.

Teorema: $P(x)$ ko'phad koeffisentlarining yig'indisi $P(1)$ ga teng.

$P(1)=a+b+c+\ldots+t+r$

Misol: Ushbu ko'phad koeffisentlari yig'indisini toping. $P(x)=4(x-1)^9+x^2(x-3)$

Yechish: $P(1)=4(1-1)^9+1^2(1-3)=0-2=-2$

Teorema: $P(x)$ ko'phadning juft darajalari hadlari koeffisentlari yig'indisi

$$\frac{P(1)+P(-1)}{2}$$ ga teng.

Teorema: $P(x)$ ko'phadning toq darajalari hadlari koeffisentlari yig'indisi

$$\frac{P(1)-P(-1)}{2}$$ ga teng.

Misol: $P(x)=2x^3(x^2-3x-1)+(x-1)^4$ ko'phadning toq darajali hadlari koeffisentlari yig'indisini toping.

Yechish: $P(1)=2\cdot1^3\cdot(1^2-3\cdot1-1)+(1-1)^4=-6$

$P(-1)=2\cdot(-1)^3\cdot((-1)^2-3\cdot(-1)-1)+(-1-1)^4=10,$ $\frac{P(1)-P(-1)}{2}=\frac{-6-10}{2}=-8$

Eslatma: $P(x)$ ko'phadni $x-a$ ifodaga bo'lgandagi qoldiq $P(a)$ ga teng. a-berilgan son.

Misol: $P(x)=2x^3-3x^2+4x-1$ ko'phadni $x-2$ ga bo'lgandagi qoldiqni toping.

Yechish: $a=2$, $P(2)=2\cdot2^3-3\cdot2^2+4\cdot2-1=11$

Eslatma: Ifodalarning eng katta yoki eng kichik qiymatlarini topishda, ba'zan taqqoshlashda ushbu tengliklardan foydalaniladi.

a) $a+b\geq2\sqrt{ab},\quad a\geq0,\quad b\geq0$

b) $p^2+q^2+r^2\geq pq+pr+qr,\quad p,q,r\in R$

Eslatma: Faraz qilaylik 1 dan n gacha barcha natural sonlar ko'paytmasi tub ko'paytuvchilarga ajratilgan bo'lsin. Yoyilma quyidagi ko'rinishda bo'ladi:

$n!=1\cdot2\cdot3\cdot\ldots\cdot n=2^a\cdot3^b\cdot5^c\cdot7^d\cdot\ldots$

Bundan shu yoyilmada birorta tub sonning, masalan 5 ning nechtaligini aniqlash so'ralgan bo'lsin.Ko'paytmadagi 5 lar soni quyidagiga teng bo'ladi:

$$k=\left[\frac{n}{5}\right]+\left[\frac{n}{5^2}\right]+\left[\frac{n}{5^3}\right]+\left[\frac{n}{5^4}\right]+...$$

Bunda $\left[\frac{n}{5}\right]$ yozuv $\frac{n}{5}$ ning butun qiymatini anglatadi. Qo'shiluvchilar qo'shiluvchining butun qismi 1 dan kichik bo'lib qolguncha olinadi.

Misol: 1 dan 1000 gacha bo'lgan barcha natural sonlar ko'paytmasini tub ko'paytuvchilarga ajratsak, ko'paytmada 3 soni nech marotaba qatnashadi?

Yechish: $1\cdot 2\cdot 3\cdot ...\cdot 1000=2^a\cdot 3^b\cdot 5^c\cdot ...$

$$k=\left[\frac{1000}{3}\right]+\left[\frac{1000}{3^2}\right]+\left[\frac{1000}{3^3}\right]+\left[\frac{1000}{3^4}\right]+\left[\frac{1000}{3^5}\right]+\left[\frac{1000}{3^6}\right]=333+111+37+12+4+1=498 \text{ ta}$$

Eslatma: Agar a va b berilgan son bo'lib, ular ushbu tengsizlikni qanoatlantirsa $a\le x\le y\le z\le t\le b$, u holda $\frac{x}{y}+\frac{z}{t}\ge 2\cdot\sqrt{\frac{a}{b}}$ ifodaning eng kichik qiymati $2\cdot\sqrt{\frac{a}{b}}$ ga teng.

Misol: Agar x,y,z va t sonlari ushbu $7\le x\le y\le z\le t\le 112$ tengsizlikni qanoatlantirsa $\frac{x}{y}+\frac{z}{t}$ ifodaning eng kichik qiymatini toping.Yechish:

$$\frac{x}{y}+\frac{z}{t}\ge 2\cdot\sqrt{\frac{7}{112}}=2\cdot\sqrt{\frac{1}{16}}=2\cdot\frac{1}{4}=\frac{1}{2} \quad \text{J:}\frac{1}{2}$$

Eslatma: Ba'zi misollarda kasr ifodalarning surat va maxrajida trigonometrik ifodalar bo'lib, bu kasrning eng katta yoki eng kichik qiymatini topish so'ralgan. Agar kasrning eng katta qiymatini topish so'ralsa surati eng katta, maxraji eng kichik bo'lishi kerakligini, agar kasrning eng kichik qiymatini topish so'ralsa surati eng kichik, maxraji eng katta bo'lishi kerakligini yodda tutish zarur.

Misol: Ifodaning eng katta qiymatini toping. $\frac{8\cos 2\alpha-5\cos 3\beta}{7+2\cos 4\gamma}$

Yechish: $\cos 2\alpha = 1,\ \cos 3\beta = -1,\ \cos 4\gamma = -1$ bo'lishi kerak. Shunda kasr eng katta qiymatga ega bo'ladi. $\frac{8\cdot 1 - 5\cdot(-1)}{7 + 2\cdot(-1)} = \frac{13}{5} = 2{,}6$.

Eslatma: Uchta va to'rtta sonlar uchun Koshi-Bunyakovskiy tengsizligi quyidagicha bo'ladi.

$$a+b+c \ge 3\sqrt[3]{abc}, \quad a+b+c+d \ge 4\sqrt[4]{abcd}$$

To'la kvadrat ifodalar.

$a^2 \pm 2ab + b^2$ ko'rinishidagi ifodani to'la kvadrat ifoda deyiladi. Chunki bu ifodalarni ikkihadning kvadrat shaklida yig'sa bo'ladi. $a^2 \pm 2ab + b^2 = (a \pm b)^2$

Misol: k ning qanday qiymatida ushbu $x^2 + 6x + k$ ifoda to'la kvadrat bo'ladi.

Yechish: **I-usul:** $x^2 + 6x + k = x^2 + 2\cdot 3x + k,\ k = 3^2 = 9$.

II-usul: $D = b^2 - 4ac = 0$ deb yechsa ham bo'ladi.

Misol: m ning qanday qiymatida ushbu $x(x+a)(x+b)(x+a+b) + 4m^2$ ifoda to'la kvadrat bo'ladi.

Yechish:

$$x(x+a)(x+b)(x+a+b) + 4m^2 = (x^2+ax+bx)(x^2+ax+bx+ab) + 4m^2 =$$

$$= (x^2+ax+bx)^2 + ab(x^2+ax+bx) + 4m^2 = (x^2+ax+bx)^2 + 2\cdot\frac{ab}{2}\cdot(x^2+ax+bx) + 4m^2$$

Bunda $4m^2 = (\frac{ab}{2})^2,\ 2m = \pm\frac{ab}{2},\ m = \pm\frac{ab}{4}$.

Misol: Tenglamani yeching. $\left(\sqrt{5}\right)^x + \left(\sqrt{12}\right)^x = \left(\sqrt{13}\right)^x$ Yechish:

Har ikkala tomonini $\left(\sqrt{13}\right)^x$ ga bo'lamiz.

$$\left(\frac{\sqrt{5}}{\sqrt{13}}\right)^x + \left(\frac{\sqrt{12}}{\sqrt{13}}\right)^x = 1, \quad \left(\sqrt{\frac{12}{13}}\right)^x = 1 - \left(\sqrt{\frac{5}{13}}\right)^x \Rightarrow \begin{cases} y_1 = \left(\sqrt{\frac{12}{13}}\right)^x & \text{grafigini yasaymiz} \\ y_2 = 1 - \left(\sqrt{\frac{5}{13}}\right)^x & \text{grafigini yasaymiz} \end{cases}$$

Yechish: bunday tenglamalardan javoblardagi oraliqlardan biror sonni olib o'rniga qo'yish kerak.

Misol: $m^2+n^2=p^2+q^2,\quad mp+nq=0,\quad mn+pq=?$

I-usul: $mp=-nq,\quad m^2p^2=n^2q^2$

$$\left.\begin{array}{l}1)\ m^2+n^2=1,\quad m^2=1-n^2\\ 2)\ q^2+p^2=1,\quad q^2=1-p^2\end{array}\right\}\Rightarrow(1-n^2)p^2=n^2(1-p)^2,\quad p^2-n^2p^2=n^2-p^2n^2$$

$p^2=n^2\Rightarrow m^2=q^2\Rightarrow$ 1) p = n, p = -n 2) m = q, m = -q

$mp+nq=0,\ qp+pq=0,\quad 2pq=0,\quad pq=0$ 1) $p=0,\ q=1$ 2) $p=1,\ q=0$

FOYDALANILGAN ADABIYOTLAR RO'YXATI

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