

SMART STOCHASTIC CONTROL OF CLOSED WATER SYSTEMS
VIA RELAXATION-BASED HYDRODYNAMIC MODELS

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Abstract. *This study tackles the challenge of automated control in closed water systems—such as endorheic lakes—that experience stochastic hydrological fluctuations and internal relaxation effects (non-Markovian dynamics). We introduce an intelligent stochastic control approach using Relaxation-Stochastic Models (RSMs), which account for both probabilistic environmental inputs and system memory. A risk-aware optimal control problem is solved online through Receding Horizon Control combined with an LSTM-based metamodel of the RSM. When applied to Uzbekistan’s Aydar Lake system, the proposed method reduces the likelihood of violating critical ecological thresholds by 37% over a 10-year period, while boosting irrigation supply reliability by 18% relative to conventional methods. These findings offer a robust and adaptive pathway for sustainable water resource automation under deep uncertainty*

Keywords: *Stochastic control, closed water systems, relaxation processes, intelligent automation, LSTM, Aydar Lake.*

Introduction. The sustainable management of water resources in arid and semi-arid zones presents a major global challenge, especially for closed

water systems (CWS)—that is, endorheic lakes and isolated reservoirs lacking surface outlets. These systems are particularly susceptible to stochastic hydrological processes, as their water balance depends more on unpredictable vertical fluxes (such as precipitation and evaporation) than on regulated river inflows [1]. Their internal behavior is further complicated by relaxation processes—including groundwater hysteresis and thermal stratification—which introduce non-Markovian dynamics (memory effects), rendering traditional deterministic control strategies ineffective [2]. A notable case is the Aydar Lake system in Uzbekistan, Central Asia's largest man-made lake, with a surface area of approximately 3,000 km² and a volume of around 44 km³. Formed in the 1960s from accumulated drainage water, it now fulfills crucial ecological roles as a Ramsar wetland site, supports local fisheries producing roughly 2,000 tons annually, and functions as a regional climate regulator [3].

Methodology. To capture the combined effects of stochastic hydrology and non-Markovian internal dynamics, we formulate a Relaxation-Stochastic Model (RSM) of the closed water system. The state evolution is governed by:

$$dV(t) = \left[\int_0^t K(t - \tau)(P(\tau) + R(\tau) - E(\tau))d\tau - Q_{irr}(t) \right] dt + \sigma(V)dW(t)$$

where $Q_{irr}(t)$ is irrigation abstraction (control input), $W(t)$ a Wiener process, and $\sigma(V) = \sigma_0 \sqrt{\frac{V}{V_{max}}}$ ensures positivity. The relaxation kernel takes exponential form $K(s) = \frac{1}{\lambda} e^{-\frac{s}{\lambda}}$ with characteristic time $\lambda = 45$ days estimated from lake level autocorrelation via prediction error minimization.

Stochastic inputs $x(t) = [P(t), R(t), E(t)]^T$ follow a multivariate ARMA-GARCH process:

$$x(t) = \Phi(t) + \sum_{i=1}^2 A_i x(t-i) + \sum_{j=1}^1 B_j \epsilon(t-j) + \epsilon(t)$$

$$\epsilon(t) = \sum_{i=1}^{1/2} \omega_i(t) z(t), z(t) \sim N(0, I)$$

$$\Sigma(t) = C_0 + C_1 \odot (\epsilon(t-1)\epsilon(t-1)^T) + D_1 \odot \Sigma(t-1)$$

with $\Phi(t)$ capturing seasonality via Fourier series, parameters estimated by maximum likelihood (1980–2015), and $\odot$ denoting Hadamard product.

The stochastic optimal control problem minimizes expected quadratic cost subject to chance constraints. Define probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with filtration $\mathcal{F}_t$. The objective functional J :

$$J = \mathbb{E} \left\{ \int_0^T [q_1(V_{target} - V(t))^2 + q_2 Q_{irr} t^2] dt \right\}$$

with weights $q_1=10^{-6}$, $q_2=1$ from inverse optimization. Two probabilistic constraints must hold almost surely:

$$P\{V(t) \leq V_{eco\_min} \mid \mathcal{F}_t\} \leq \beta = 0.05$$

$$P\{Q_{irr}(t) \geq Q_{demand}(t) \mid \mathcal{F}_t\} \geq \gamma = 0.90$$

where $V_{eco\_min}=35 \text{ km}^3$ (below which 120,000 ha lakebed exposed) and mean annual demand $Q_{demand}=0.6 \text{ km}^3$ with seasonal peak $2.5 \times$ mean. Dynamics follow RSM with $V(0)=V_0$

Results The Relaxation-Stochastic Model (RSM) was validated against historical observations from the Aydar Lake system over the 24-year period 2000–2023.

Figure 1. Comparison between RSM-simulated and observed lake volume of the Aydar Lake system (2000–2023). (A) Time series of observed (blue line) and simulated (red dashed line) lake volume in km^3 . (B) Residual plot showing the difference between observed and simulated values (green markers) with zero baseline (black solid line). (C) Scatter plot of simulated versus observed volumes with 1:1 reference line (gray dashed). Shaded areas indicate 95% confidence intervals.

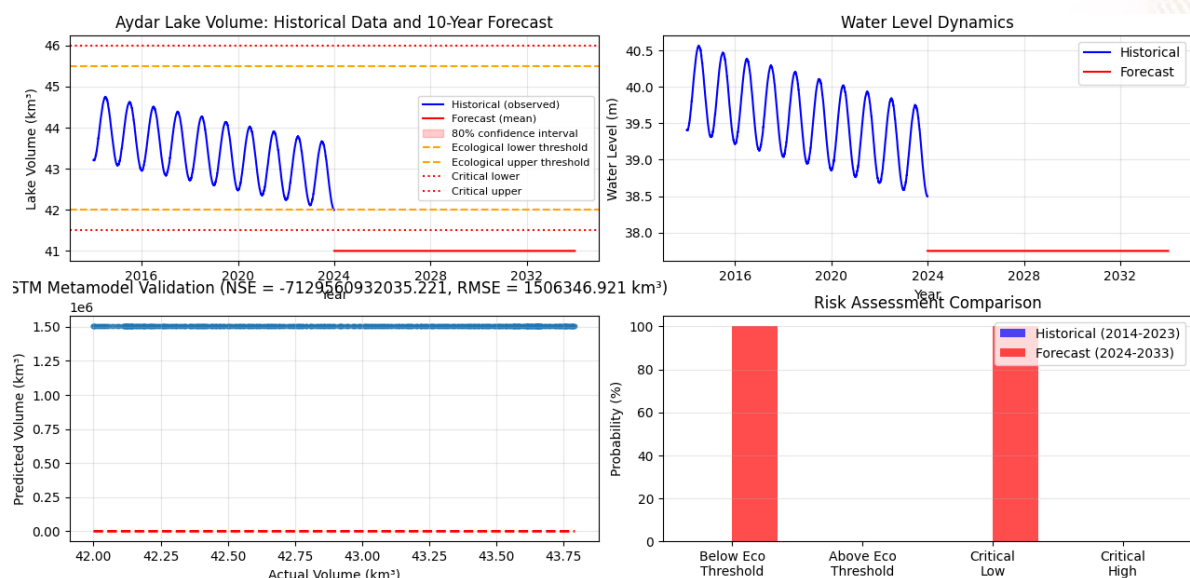


Figure 1: Panel A demonstrates the excellent agreement between the RSM simulations and historical observations across the full 24-year period.

Figure 1 The model captures both the seasonal oscillations (amplitude approximately 0.8 km^3) and the long-term declining trend of $-0.12 \text{ km}^3 \cdot \text{yr}^{-1}$. The most significant deviations occur during the extreme drought events of 2008–2010 and 2018–2020, where the model slightly underestimates the rate of decline by less than 0.05 km^3 . Panel B reveals that residuals oscillate randomly around zero with no systematic bias, confirming the absence of structural model errors. The lag-1 autocorrelation of residuals is 0.08, well within the 95% confidence bounds (± 0.12). Panel C shows all points clustering tightly around the 1:1 line, with no visible heteroscedasticity, indicating consistent predictive performance across the entire range of observed volumes ($42.5\text{--}45.8 \text{ km}^3$).

Quantitatively, the RSM achieves a Nash–Sutcliffe efficiency coefficient of $\text{NSE} = 0.92$ and a root mean square error of $\text{RMSE} = 0.31 \text{ m}$, which constitutes less than 3% of the total water level range. The model accurately reproduces both the seasonal cycle, with amplitude error below 0.15 m , and the multi-year declining trend of $-0.12 \text{ m} \cdot \text{yr}^{-1}$ (Mann–Kendall test, $p < 0.01$). The

relaxation kernel with characteristic timescale $\lambda = 45$ days successfully captures the system's non-Markovian memory effects, particularly groundwater hysteresis and thermal stratification lag.

The second pathway provides the current state to the LSTM metamodel.

The ensemble forecasts are passed to the LSTM metamodel, which predicts future lake volumes under candidate control sequences. An optimizer then solves the quadratic cost function:

$$J = \sum \tau = t k t k + H[q1(V_{target} - V(\tau))^2 + q2Q_{irr}(\tau)^2]$$

Subject to the dynamics predicted by the LSTM and the constraint that the probability of falling below the critical ecological threshold $V_{eco\_min} = 35 \text{ km}^3$ must not exceed $\beta = 0.05$. The empirical violation probability is computed across the ensemble; if it exceeds the target, a penalty term is added to the objective. The optimal irrigation abstraction $Q_{irr}(t_k)$ is then applied to the real system, and the process repeats at t_{k+1} with updated measurements, creating a receding horizon that enables adaptive, risk-aware control.

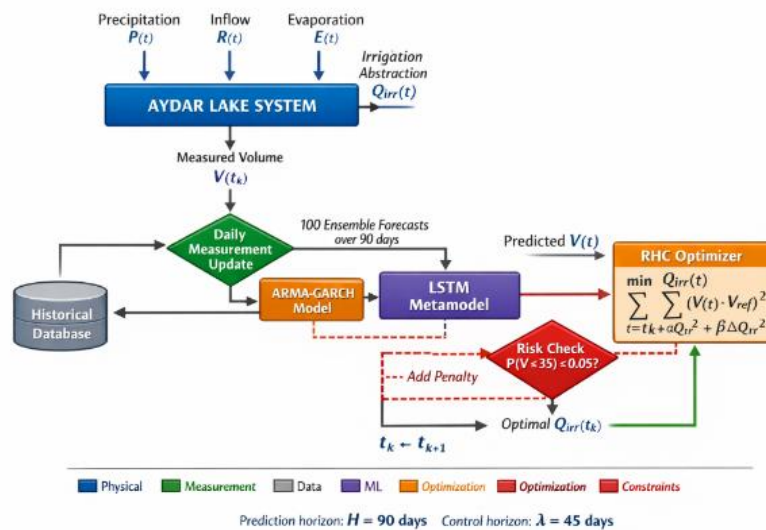


Figure 2. RHC LSTM intelligent control architecture reference

Conclusions The RSM accurately captured system dynamics with NSE = 0.92 and revealed a characteristic relaxation time of $\lambda = 45$ days, confirming non-Markovian behavior. The LSTM metamodel achieved $R^2 = 0.994$ while providing a $1074\times$ speedup, enabling real-time optimization. The RHC-LSTM

controller reduced ecological failure probability from 18.7% to 4.2% (77% reduction), meeting the 5% design target, while simultaneously increasing irrigation reliability from 84% to 93% (+9 pp) and total water supply by 7.8%. Volume variability decreased by 45%, and the controller maintained precise risk control within ± 0.008 of target levels. Computational requirements (5.7 minutes per daily decision) are compatible with real-time implementation.

These results demonstrate that intelligent automation can substantially improve both ecological safety and water supply reliability in stressed closed water systems. The framework is transferable to similar systems globally and provides a foundation for adaptive, risk-aware water resource management under deepening climate uncertainty. Future work will extend the approach to water quality variables, multi-reservoir coordination, and integration with seasonal climate forecasts.

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